UNIV 2014 Assignment #1

Deadline: March 20, 2019

1) A=$\begin{bmatrix} 4 & 1 & -2 \\ 3 & 1 & -1 \end{bmatrix}$, B^T=$\begin{bmatrix} 1 & 3 & -2 \\ 5 & -1 & 4 \end{bmatrix}$, C^T=$\begin{bmatrix} 3 & -1 & 0 \\ 1 & 7 & 0 \end{bmatrix}$
   a) Find B, C, and A^T.
   b) Find AB.
   c) Find AC.
   d) Find B+C.

2) Is the following statement false in linear algebra? If AB=AC, then B=C. Why? (Hint: You can use the matrices of question 1.)

3) (Elementary Matrix Operations) Let A be a three by three matrix. After 4 operations, I (identity matrix) is obtained from A.
   $R_1 \leftrightarrow R_2$
   $5R_2 \rightarrow R_2$
   $2R_2 + R_3 \rightarrow R_3$
   $-R_1 + R_2 \rightarrow R_2$
   a) Find A.
   b) Find A^{-1}.

4) (Elementary Matrix Operations)
   A= $\begin{bmatrix} 2 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}$
   a) Given the matrix A, find its inverse if it is invertible.
   b) Also find the k elementary matrices such that $E_1 E_2 \ldots E_k A = I$.

5) (Elemantary Matrix Operations & Solving linear algebraic equations)
   A= $\begin{bmatrix} 1 & 3 & -2 \\ -1 & -5 & 3 \\ 2 & -8 & c \end{bmatrix}$
   a) Find a row-reduced echolon matrix that is row-equivalent to A.
   b) Given Ax=b, and c=3, b=$\begin{bmatrix} 1 \\ -1 \\ a \end{bmatrix}$. Find the value of a which makes the system consistent (there is a solution.) Find the solution for that a.
   c) Given Ax=b, b=$\begin{bmatrix} 1 \\ -1 \\ a \end{bmatrix}$, find possible values of a and c for which the system has a unique solution.

6) (Solving linear algebraic equations)
   A=$\begin{bmatrix} 1 & -2 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$
   C=$\begin{bmatrix} 1 & -2 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$
   a) Find the homogenous, particular and the general solution of AX=b, if b=$\begin{bmatrix} 3 \\ 5 \end{bmatrix}$.
   b) Find the solution of CX=$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.
   c) Find the solution of $\begin{bmatrix} A \\ C \end{bmatrix}$X=$\begin{bmatrix} b \\ 0_{2X1} \end{bmatrix}$. 0_{2X1} means a 2 by 1 zero matrix.

7) [Determinant and inverse matrix] Find the inverse of matrix A using the cofactors.

A=$\begin{bmatrix} -1 & 0 & 0 \\ -1 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

8) [Cramer's Rule] Solve the following systems for y using Cramer's rule.

   a) x+ 2y =2
      3x +4y = 1
   b) 21x+7y-4z=20
      7x+13y+13z=17
      -4x+3y+14z=12

9) [Determinant] Use row operations to verify that the determinant of a 3 by 3 Vandermonde matrix is (b-a)(c-a)(c-b).

A=$\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$